NONSTEADY FILTRATION IN A SEMIINFINITE TWO-LAYERED MEDIUM

A. A. Belyi, R. M. Éigeles,
and A. F. Él'kind

UDC 532.54

The system of equations of nonsteady filtration of a uniform liquid in a two-layered medium, the first layer of which is biporous while the second is purely porous, is solved. The analogous problem is solved for the case of filtration of a suspension or a colloidal solution.

Problems of the nonsteady filtration of uniform liquids or suspensions are of interest in the study of processes of transfer in various natural and aritifical nonuniform porous media. Such media often have a layered character, with the layers differing in a number of physicochemical parameters: permability, compressibility, piezoconductivity, and the character of the porosity. In the cases when each of the layers is described by a model of a simple porous medium, the filtration equation is analogous to a diffusion equation, and filtration in multilayered media can be described by methods of diffusional kinetics [1].

Situations in which at least one of the layers has a complicated porous structure, when cracks can be taken into account in addition to pores, are of particular interest. Examples of such media are fissured-porous rocks [2] and anisotropic membranes used in chemical engineering for the separation of mixtures [3]. These layers can develop near the surface of various materials disturbed by mechanical action.

A layer can be described mathematically by the model of a biporous medium. In the process of filtration in such media, mass exchange occurs between pores of different types, which has a considerable influence on the character of the redistribution of pore pressure in the system. Allowance for this exchange requires modification of the filtration equation and specifies the form of the boundary conditions.

The presence of such a layer (described by the model of a biporous medium) in a multilayered medium has considerable influence on the character of the pore pressure in the ordinary porous media of other layers over time intervals comparable with the characteristic times of transitional processes in the biporous medium.

In the present paper we investigate the simplest model of filtration in a two-layered medium, one of the layers of which (the first) is described by a biporous model. The layers are arranged horizontally. Let the second layer, occupying the region of space $x_0 \leq x < \infty$, be purely porous with a permeability and piezoconductivity k_2 and $\varkappa_2$, respectively. We designate the liquid pressure in layer II as $P_\text{p}(x, t)$. Then one-dimensional filtration in this layer is described by the usual equation of piezoconductivity [4]

$$\frac{\partial P_\text{p}}{\partial t} = \varkappa_2 \frac{\partial^2 P_\text{p}}{\partial x^2}. \tag{1}$$

The first layer of thickness x_0 consists of a biporous medium [2, 5], the permeable channels of which consist of cracks (macropores) and pores. The equation of one-dimensional filtration of liquid in the pores of such a medium, as is well known [2, 5], has the form

$$\frac{\partial P_\text{p}}{\partial t} = \eta \frac{\partial^3 P_\text{p}}{\partial x^2 \partial t} + \varkappa_1 \frac{\partial^2 P_\text{p}}{\partial x^2}, \tag{2}$$

where $\eta \simeq (k_1/k_{1\text{p}})\ell^2$.

The first term on the right side of Eq. (2) allows for the internal flows of the filtered liquid between pores of poorly permeable blocks and the cracks separating blocks. Because of the low permeability of the porous blocks, liquid exchange between points adjacent to them is small compared with the flow from cracks into the blocks.

All-Union Scientific-Research Institute of Borax Technology, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 52, No. 1, pp. 25-32, January, 1987. Original article submitted October 22, 1985.

 0022-0841/87/5201-0018$12.50

Thus, equalization of the pore pressures at adjacent points of a biporous medium is possible only through liquid exchange between blocks and cracks and liquid movement through cracks. This leads to the fact that jumps in pore pressure in such a medium are not washed out instantly but die out with time.

We shall assume that the pressure at the boundary of the half-space (x = −0) is kept constant and equal to P_1. The pressure in the medium at the initial time is P_2. The problem of determining $P_c(x, t)$ and $P_p(x, t)$ comes down to the solution of the system of equations (1) and (2) with the following initial and boundary conditions:

$$P_p(x, 0) = P_2, \; P_c(x, 0) = P_2; \tag{3}$$

$$P_p(\infty, t) = P_2. \tag{4}$$

At the initial time t = 0 at the boundary of layer I at the point x = 0 there is a jump in pressure, equal to $(P_1 - P_2)$; as was shown in Refs. 2 and 5, the law of damping of jumps in pore pressure has an exponential character, and at the time t this jump will be $(P_1 - P_2)\exp(-\varkappa_1 t/\eta)$.

Thus, the pore pressure of the liquid below the boundary is

$$P_c(+0, t) = P_1 + (P_2 - P_1)\exp(-\varkappa_1 t/\eta). \tag{5}$$

Equation (5) determines the boundary condition for Eq. (2). The characteristic delay time $\tau = \eta/\varkappa_1$ is determined by the properties of the biporous medium and the viscosity of the liquid being filtered.

At the point x_0 at the boundary between the layers we have the condition of equality of the pressures $P_c(x_0 - 0, t)$ and $P_p(x_0 + 0, t)$ and the liquid flows. We have $P_c(x_0 - 0, t) = P_p(x_0 + 0, t)$ and

$$\frac{k_1}{\mu}\left(\frac{\partial P_c}{\partial x} + \frac{\eta}{\varkappa_1}\frac{\partial^2 P_c}{\partial x \partial t}\right)\Bigg|_{x=x_0-0} = \frac{k_2}{\mu}\left(\frac{\partial P_p}{\partial x}\right)\Bigg|_{x=x_0+0}, \tag{6}$$

where μ is the viscosity of the liquid being filtered.

To seek a solution of the system of equations (1), (2), we take

$$P_c(x, t) = P_2 + U_c(x, t), \quad P_p(x, t) = P_2 + U_p(x, t). \tag{7}$$

Converting from $U_c(x, t)$ and $U_p(x, t)$ to their Laplace transforms

$$U_c(x, s) = \int_0^\infty U_c(x, t)\exp(-st)\,dt,$$
$$U_p(x, s) = \int_0^\infty U_p(x, t)\exp(-st)\,dt, \tag{8}$$

we obtain a system of equations for determining $U_c(x, s)$ and $U_p(x, s)$:

layer I

$$\frac{d^2 U_c(x, s)}{dx^2} - \frac{sU_c(x, s)}{\varkappa_1 + s\eta} = 0, \tag{9}$$

$$U_c(0, s) = \frac{(P_1 - P_2)\varkappa_1}{s(\varkappa_1 + s\eta)}; \tag{10}$$

layer II

$$\frac{d^2 U_p(x, s)}{dx^2} - \frac{sU_p(x, s)}{\varkappa_2} = 0, \tag{11}$$

$$U_p(\infty, s) = 0, \tag{12}$$

$$U_c(x_0, s) = U_p(x_0, s),$$
$$k_1\left(\frac{\partial U_c(x, s)}{\partial x}\right)_{x=x_0-0}\left(1 + \frac{\eta s}{\varkappa_1}\right) = k_2\left(\frac{\partial U_p(x, s)}{\partial x}\right)_{x=x_0+0}. \tag{13}$$

Solving this system for $U_c(x, s)$ and $U_p(x, s)$, we have

$$U_c(x, s) = \frac{(P_1 - P_2)\varkappa_1}{s(\varkappa_1 + s\eta)}\left\{\exp\left(-\left(\frac{s}{\varkappa_1 + s\eta}\right)^{1/2} x\right) + \right.$$
$$+ \exp\left(-\left(\frac{s}{\varkappa_1 + s\eta}\right)^{1/2}(2x_0 - x)\right)\left.\frac{\varkappa_2^{1/2} - a\varkappa_1(\varkappa_1 + s\eta)^{-1/2}}{\varkappa_2^{1/2} + a\varkappa_1(\varkappa_1 + s\eta)^{-1/2}}\right\} \times \tag{14}$$
$$\times \left\{1 + \frac{\varkappa_2^{1/2} - a\varkappa_1(\varkappa_1 + s\eta)^{-1/2}}{\varkappa_2^{1/2} + a\varkappa_1(\varkappa_1 + s\eta)^{-1/2}}\exp\left(-2\left(\frac{s}{\varkappa_1 + s\eta}\right)^{1/2} x_0\right)\right\}^{-1},$$

$$U_p(x, s) = \frac{2(P_1 - P_2)\varkappa_1}{s(\varkappa_1 + s\eta)} \times$$
$$\times \left\{\frac{\varkappa_2^{1/2}\exp\left(-\left(\frac{s}{\varkappa_1 + s\eta}\right)^{1/2} x_0 + \left(\frac{s}{\varkappa_2}\right)^{1/2}(x_0 - x)\right)}{\varkappa_2^{1/2} + a\varkappa_1(\varkappa_1 + s\eta)^{-1/2}}\right\} \times \tag{15}$$
$$\times \left\{1 + \frac{\varkappa_2^{1/2} - a\varkappa_1(\varkappa_1 + s\eta)^{-1/2}}{\varkappa_2^{1/2} + a\varkappa_1(\varkappa_1 + s\eta)^{-1/2}}\exp\left(-2\left(\frac{s}{\varkappa_1 + s\eta}\right)^{1/2} x_0\right)\right\}^{-1},$$

where $a = k_2/k_1$. Equations (14) and (15) enable one to obtain the pressure distribution in a purely porous two-layered medium if one takes η to zero in them, which actually means the absence of liquid exchange between the cracks and pores of the blocks. The case of steady filtration in a biporous medium coincides with the case of steady filtration in a porous medium, since in this case we have in mind times much longer than the characteristic delay time, $t \gg \eta/\varkappa_1$. Conversion to the inverse transforms

$$U_c(x, t) = (2\pi i)^{-1}\int_{b-i\infty}^{b+i\infty} U_c(x, s)\exp(st)\,ds, \qquad (b > 0) \tag{16}$$
$$U_p(x, t) = (2\pi i)^{-1}\int_{b-i\infty}^{b+i\infty} U_p(x, s)\exp(st)\,ds,$$

and the use of Eq. (7) yields the unknown pressure distribution in the medium under consideration. However, an inverse Laplace transformation of (14) and (15) through Eqs. (16) is only possible numerically in the general case. If we must establish the asymptotic behavior of the functions $U_c(x, t)$ and $U_p(x, t)$ at $t \ll \eta/\varkappa_1$, i.e., for times less than the characteristic time of the transitional processes, then we obtain

$$U_c(x, t) \simeq (P_1 - P_2)(1 - \exp(-\varkappa_1 t/\eta)) \times \frac{\exp(-x\eta^{-1/2}) + \exp((x - 2x_0)\eta^{-1/2})}{1 + \exp(-2x_0\eta^{-1/2})}, \tag{17}$$

$$U_p(x, t) \simeq \frac{2(P_1 - P_2)\varkappa_1\exp(-x_0\eta^{-1/2})}{\eta(1 + \exp(-2x_0\eta^{-1/2}))}\int_0^t \operatorname{erfc}\frac{(x - x_0)}{2(\varkappa_2\tau)^{1/2}}\,d\tau, \tag{18}$$

where $\operatorname{erfc}(z) = 2\pi^{-1/2}\int_z^\infty \exp(-g^2)\,dg$. From (17) it is seen that $U_c(x, t)$ depends on the thickness of the first layer and does not depend on the permeability or piezoconductivity of the second layer, i.e., for times $t < \eta/\varkappa_1$ only the boundary between the layers is "felt." Equations (17) and (18) were obtained in the limit of small times $t \ll \eta/\varkappa_1$, but this dependence of $U_c(x)$ on time is retained up to times on the order of $\eta/\varkappa_1$. At large distances from x_0, i.e., for $(x - x_0)/2(\varkappa_2\tau)^{1/2} \gg 1$, from (18) we find

$$U_p(x, t) \simeq \frac{2(P_1 - P_2)\exp(-x_0\eta^{-1/2} - (x - x_0)^2(4\varkappa_2 t)^{-1})\varkappa_1 t}{(1 + \exp(-2x_0\eta^{-1/2}))\pi^{1/2}(x - x_0)(4\varkappa_2 t)^{-1/2}\eta}. \tag{19}$$

An investigation of filtration in the first layer using Eq. (2) is justified in the case when the characteristic times of the given problem are on the order of the characteristic delay time of the processes of pressure equalization in the biporous medium, i.e., when the relation $x_0^2 \lesssim \eta$ is satisfied. This inequality actually means that $x_0 \lesssim \ell(k_1/k_{1p})^{1/2}$. Moreover, it is necessary that the condition $x_0 \gg \ell$ be satisfied.

Thus, the condition for the problem under consideration to be well-posed is written in the form

$$1 \ll \frac{x_0}{l} \leqslant (k_1/k_{1p})^{1/2}.$$

Consequently, in the case when x_0/ℓ is a sufficiently large finite quantity, the given problem, using a boundary condition of type (5) and Eq. (2), is well-posed.

If $x_0 \to \infty$, then from the inequality obtained it follows that $\eta \to \infty$, so that $\varkappa_1/\eta \to 0$. Therefore, to describe liquid filtration in a semiinfinite fractured-porous medium one must use Eq. (2) for the pressure $P_c^0(x, t)$ in the cracks and the boundary condition from the problem of filtration in a porous medium [2]. With such a statement of the problem, the solution is easily found in the s-representation:

$$P_c^0(x, s) = \frac{P_2}{s} + \frac{P_1 - P_2}{s} \exp\left(-\sqrt{\frac{s}{s\eta + \varkappa_1}}\, x\right).$$

Let us consider the asymptotic behavior of the pressure $P_c^0(x, s)$ for $s \gg \varkappa_1/\eta$ (i.e., $t \ll \eta/\varkappa_1$). We expand the solution found in a series in powers of $1/s$, confining ourselves to the first two terms of the series. Converting term by term to the inverse transforms [6], we obtain

$$P_c^0(x, t) \approx P_2 + (P_1 - P_2)\exp(-x\eta^{-1/2})\left\{1 + \frac{\varkappa_1 x}{2\eta^{3/2}}\, t\right\}.$$

From this it is seen that the pressure in a semiinfinite fractured-porous medium in the region of small times near the boundary of the plate differs little from P_1 in comparison with the pressure in the pores of a fractured-porous medium of finite thickness (17), (7).

The delay times $\eta/\varkappa_1$ can be considerable for certain parameters of the biporous medium. For example, in the investigation of geological structures, which are natural biporous media, the delay times reach 10^4 sec.

For practical purposes it is desirable to know the time dependence of the flow of liquid entering the first layer. This flow is proportional to the derivative of the pressure in the cracks and, as was shown in [5], is

$$Q = -\frac{k_1}{\mu}\left(\frac{\partial P_c(x, t)}{\partial x} + \frac{\eta}{\varkappa_1}\frac{\partial^2 P_c(x, t)}{\partial x \partial t}\right)_{x=0}. \tag{20}$$

For times $t \ll \eta/\varkappa_1$ we find the specific flow rate of the filtrate through the boundary of half-space from (17) and (20):

$$\int_0^t Q dt = \frac{k_1 (P_1 - P_2)(1 - \exp(-2x_0\eta^{-1/2}))\, t}{\mu\eta^{1/2}(1 + \exp(-2x_0\eta^{-1/2}))}. \tag{21}$$

In contrast to a purely porous two-layered medium, where at small times

$$U_c(x, t) \simeq (P_1 - P_2)\,\mathrm{erfc}\left(\frac{x}{2(\varkappa_1 t)^{1/2}}\right),$$

$$U_p(x, t) \simeq (P_1 - P_2)\,\mathrm{erfc}\left(\frac{x - x_0}{2(\varkappa_2 t)^{1/2}}\right) \text{ and } Q \simeq \frac{k_1 (P_1 - P_2)}{\mu(\pi\varkappa_1 t)^{1/2}},$$

it is seen from (17)-(21) that the presence of a biporous surface layer leads to different time dependences of the pressure and the flow of the filtrate. If the characteristic delay times of the process of increase in pore pressure in the biporous medium are comparable with the times of experimental observation of the filtration process, than the functions obtained above should be used in the theoretical description of the filtration laws.

The study of nonsteady filtration of suspensions of colloidal solutions in layered media is of the highest practical interest. A qualitative feature of this process consists in the effective decrease in the permeability of the porous medium due to the depositing on its surface of particles of the disperse phase of the solution being filtered. Particles of the disperse phase covering pores of the surface layer prevent the penetration of the filtrate into the medium. The decrease in the permeability of the porous medium is equivalent to the appearance of an effective time-variable flow I(t) of liquid reflected from the surface $x = 0$.

In reality, in the filtration of a suspension, a layer of disperse particles, the thickness of which grows with time, forms at the surface of the porous medium. A description of the process of filtration of a uniform liquid in this layer within the framework of the known equations (of the type of the piezoconductivity equation) is difficult, since the assumptions that the deformation has an elastic character and there is little compressibility are not satisfied, generally speaking, for a layer of disperse particles. In addition, this layer is characterized by a porosity and permeability that vary with depth and with time, which also complicates the analytical investigation of the process. In the majority of cases in the filtration of suspensions, however, the thickness of the layer of disperse particles remains small in comparison with the characteristic size of the porous specimens under investigation, and the problem under consideration can be reduced to finding the correct boundary condition effectively describing the decrease in the permeability of the porous medium in the process of the depositing of disperse particles on its surface.

We designate the flow incident on the boundary of the half-space as I_0 and the reflected flow as I(t). We assume that the flow of filtrate entering the half-space is described by an exponential dependence on time (which is often observed experimentally [3]):

$$I_0 - I(t) = I_0 \exp(-\alpha t).$$

(Below we give the calculations allowing one to obtain this time dependence of the filtrate flow.)

Thus, the problem of the filtration of a suspension or colloidal solution in a layered semiinfinite medium is analogous to the earlier problem of the filtration of a uniform liquid, but with a boundary condition different from (5):

$$I_0 \exp(-\alpha t) = -\frac{k_1}{\mu}\left(\frac{\partial P_c}{\partial x} + \frac{\eta}{\varkappa_1}\frac{\partial^2 P_c}{\partial x \partial t}\right)_{x=+0}.$$

Integrating this equation under the condition $(\partial P_c/\partial x)_{x=+0} = 0$ at t = 0, we finally obtain the boundary condition in the form

$$\left(\frac{\partial U_c}{\partial x}\right)_{x=+0} = -\frac{I_0 \mu}{k_1(1 - \alpha\eta/\varkappa_1)}[\exp(-\alpha t) - \exp(-\varkappa_1 t/\eta)], \tag{22}$$

where $U_c(x, t)$ is also determined from the first of Eqs. (7). The flow I_0 is determined from the solution of the problem of filtration of a uniform liquid in a semiinfinite two-layered medium. Obviously, $I_0 = Q_{t=0}$, where Q is found from Eq. (20).

Finding the functions $P_c(x, t)$ and $P_p(x, t)$ in the case of the filtration of a suspension or a colloidal solution comes down to the solution of the system of equations (1) and (2) with the boundary condition (22) and the matching conditions (6). The solution of this system with the condition (22) is fully analogous to its solution with the condition (5), and we give the first term of the asymptotic expansion of $U_c(x, t)$ in the region of small times $t \ll \eta/\varkappa_1$:

$$U_c(x, t) \simeq \frac{I_0 \mu \eta^{1/2}}{k_1(1 - \alpha\eta/\varkappa_1)}[\exp(-\alpha t) - \exp(-\varkappa_1 t/\eta)]\left[\frac{\exp(-x\eta^{-1/2}) + \exp(-(2x_0 - x)\eta^{-1/2})}{1 - \exp(-2x_0\eta^{-1/2})}\right]. \tag{23}$$

From (23) it is seen that a decrease in the permeability of the medium has considerable influence on the pressure distribution, particularly if $\alpha \gg \varkappa_1/\eta$, when $U_c(x, t) \approx \varkappa_1/(\alpha\eta) \ll 1$, and it follows from (7) that the pressure in layer I hardly increases. The flow of filtrate in this layer, as follows from (20) and (23), is

$$Q(x, t) \simeq I_0 \exp(-\alpha t)\frac{\exp(-x\eta^{-1/2}) - \exp(-(2x_0 - x)\eta^{-1/2})}{1 - \exp(-2x_0\eta^{-1/2})}. \tag{24}$$

At x = 0, in accordance with the boundary condition, from (24) we obtain $Q(0, t) = I_0 \cdot \exp(-\alpha t)$. The use of this boundary condition in the treatment of the experimental results on the filtration of suspensions given in [7] yields good agreement between the calculated and experimental functions.

For a qualitative discussion of the proposed boundary condition, let us consider the process of silting up (choking) of a porous medium by particles of the disperse phase of the suspension or colloidal solution being filtered. We shall assume that the particles do not penetrate into pores of the medium but form on its surface a layer characterized by a particle surface density n(t). (Actually, particles may penetrate into the porous medium, but this is expressed quantitatively only in a renormalization of the corresponding coefficients in the functions obtained below.) The variation of the permeability of the medium can be estimated, knowing the time dependence of the surface density of deposited particles. To determine the function n(t) one must solve the problem of particle sedimentation in an external field [8] with allowance for processes taking place at the boundary of the half-space. For this purpose we consider Smolukhovskii's equation

$$\frac{\partial \rho}{\partial t} = D\frac{\partial^2 \rho}{\partial x^2} - c\frac{\partial \rho}{\partial x}. \tag{25}$$

Following [9], we write the boundary conditions for this problem in the form

$$\frac{dn}{dt} = D\left(\frac{\partial \rho}{\partial x}\right)_{x=0} - c\rho(0,\ t), \tag{26}$$

$$\alpha n = \beta\rho(0,\ t) + c\rho(0,\ t) - D\left(\frac{\partial \rho}{\partial x}\right)_{x=0}, \tag{27}$$

where n is the surface concentration; $\alpha = \lim\limits_{\tau\to 0} q_1/\tau$, $\beta = \lim\limits_{\tau\to 0} q_2/\tau$, while q_1 and q_2 are the probabilities of transitions from the surface x = 0 into the solution and the reverse, respectively, in a time τ. The boundary conditions (26) and (27) have the meaning of the equations of material balance of the disperse phase at the surface of the porous medium. Taking the initial particle distribution in the medium as uniform [$\rho(x,\ 0) = \rho_0$] and taking n(0) = 0, we use the operator method to solve (25). Converting from ρ(x, t) and n(t) to their Laplace transforms, by analogy with (8) we find the solution of (25) after simple calculations:

$$\rho(x,\ s) = -\rho_0\left(\frac{c}{s} + \frac{\beta}{\alpha+s}\right)\left(\frac{c}{2} + \left(sD + \frac{c^2}{4}\right)^{1/2} + \frac{\beta s}{\alpha+s}\right)^{-1}\exp\left[\left(-\left(\frac{s}{D} + \frac{c^2}{4D^2}\right)^{1/2} + \frac{c}{2D}\right)x\right] + \frac{\rho_0}{s}. \tag{28}$$

From (26)-(28) we obtain the expression for the surface density of deposited particles:

$$n(s) = \frac{\beta\rho_0}{s(\alpha+s)} - \rho_0\left(\frac{c}{s} + \frac{\beta}{\alpha+s}\right)\ \beta\left[\left(\frac{c}{2} + \left(sD + \frac{c^2}{4}\right)^{1/2}\right)(\alpha+s) + \beta s\right]^{-1}. \tag{29}$$

The second term on the right side of (29) is much smaller in absolute value than the first for $t \ll 4D/c^2$. Neglecting the second term at small times, we obtain

$$n(s) = \frac{\beta\rho_0}{s(\alpha+s)}. \tag{30}$$

Conversion to the inverse transform yields the unknown kinetics of growth of the surface density of particles at times $t \ll 4D/c^2$. From (30), by analogy with (16), we finally find

$$n(t) = \frac{\beta\rho_0}{\alpha}(1 - \exp(-\alpha t)). \tag{31}$$

Assuming that the flow of filtrate I(t) reflected from the surface x = 0 of the medium is proportional to the particle surface density n(t) (it is assumed that a pore is completely stopped up by one particle [10]), we arrive at the boundary condition (22) sought. For low velocities of transport of particles of the disperse phase, the characteristic time $\tau = 4D/c^2$ exceeds the experiment time, and Eq. (31) and hence the boundary condition (22) remain valid in the entire range of observation times.

NOTATION

P_c, liquid pressure in pores of the biporous layer, N/m^2; P, U, liquid pressures, N/m^2; $\varkappa_1$, piezoconductivity of the biporous medium, m^2/sec; k_1, k_{1p}, permeabilities of the system of cracks and pores, m^2; η, fracturing parameter, m^2; ℓ, average size of a porous block, m; I, Q, specific flows of liquid, m/sec; ρ, volumetric density of particles in the solution, kg/m^3; D, diffusion of particles, m^2/sec; c, velocity of transport of particles in the solution being filtered, m/sec; x, coordinate, m; t, time, sec; s, parameter of the Laplace transformation, sec^{-1}.

LITERATURE CITED

1. V. K. Egerev, Diffusional Kinetics in Stationary Media [in Russian], Moscow (1970).
2. G. K. Barenblatt and Yu. P. Zheltov, Dokl. Akad. Nauk SSSR, 132, No. 3, 545-548 (1960).

3. Yu. I. Dytnerskii, Inverse Osmosis and Ultrafiltration [in Russian], Moscow (1978).
4. A. Ban, A. F. Bogomolova, V. A. Maksimov, et al., Influence of the Properties of Rocks on the Movement of Liquid in Them [in Russian], Moscow (1962).
5. G. I. Barenblatt, Yu. P. Zheltov, and I. N. Kochina, Prikl. Mat. Mekh., 24, No. 5, 852-864 (1960).
6. G. Doetsch, Guide to the Applications of the Laplace and Z-Transforms, 2nd Ed., Van Nostrand—Reinhold, New York (1971).
7. E. I. Korol'ko, R. M. Éigeles, M. I. Lipkes, and L. K. Mukhin, Neft. Khoz., No. 9, 37-39 (1979).
8. S. Chandrasekhar, Stochastic Problems in Physics and Astronomy, Am. Inst. Phys., New York (1943).
9. A. A. Belyi, A. A. Ovchinnikov, and S. F. Timashev, Zh. Fiz. Khim., 53, No. 4, 948-952 (1979).
10. V. A. Zhuzhikov, Filtration [in Russian], Moscow (1980).

MODEL OF THERMAL DESTRUCTION OF MATERIAL SUBJECTED TO ONE-SIDED HEATING

G. A. Frolov, V. V. Pasichnyi,
Yu. V. Polezhaev, and A. V. Choba

UDC 536.212.3:629.7.021.7

The article presents a model of destruction establishing a correlation between the temperature field and the rate of destruction of the material.

It was shown in [1, 2] that under conditions of nonsteady heating with linear entrainment of the material, the path traversed by the isotherm in the range of heating time $\tau_T < \tau < \tau_\delta$ can be calculated by the formula

$$\Delta^* = K\sqrt{a}\,(\sqrt{\tau} - \sqrt{\tau_\xi}), \qquad (1)$$

and its speed by the expression

$$V_{\Theta^*} = \frac{K\sqrt{a}}{2\sqrt{\tau}}. \qquad (2)$$

According to [2], $K \neq 0$ even with $\Theta^* = 1$, it can therefore be seen from (2) that the speed of the isotherm whose temperature is equal to the surface temperature at the instant of onset of linear entrainment may exceed the speed at which the surface itself moves, and that is in contradiction to the generally accepted model of heating.

However, under real conditions the destruction of the material begins before the surface temperature becomes established, and the temperature of the onset of linear entrainment may be substantially lower than its quasisteady value. For instance, linear entrainment of quartz glass ceramics with $(\alpha/c_p)_0 \sim 3.3$ kg/(m²·sec) begins approximately at 2000°K whereas the quasisteady surface temperature under such conditions of heating is ~2500°K. In the subsonic jet of an electric-arc heater ($(\alpha/c_p)_0 \sim 1.0$ kg/(m²·sec)) the temperature of the onset of entrainment is 200-300°K higher, but the surface temperature attains 2800°K, too [3].

The results of calculations [4] showed that the process of establishing the quasisteady rate of destruction of the surface is not determined by the nature of flow in the film of melt but basically by the temperature distribution inside the solid. It follows from (2) that the speed of the isotherm corresponding to the temperature of the onset of entrainment of mass from the surface decreases from the instant τ_y within the time τ_V in proportion to $1/\sqrt{\tau}$. On the other hand, the rate of destruction of the surface in that period increases from 0 to V_∞. Assuming that the temperature field determines the nature of the change of the rate of entrainment, we represent the process in question in the form of a diagram (Fig. 1).

Institute of Materials Science, Academy of Sciences of the Ukrainian SSR, Kiev. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 52, No. 1, pp. 33-37, January, 1987. Original article submitted October 29, 1985.

 0022-0841/87/5201-0024$12.50